Scripta METALLURGICA et MATERIALIA
Vol. 25, pp. 761-766, 1991
Printed in the U.S.A.

ON THE RELAXATION OF A MISMATCHING SPHEROID BY PRISMATIC LOOP PUNCHING

David C. Dunand and Andreas Mortensen
Department of Materials Science and Engineering
Massachusetts Institute of Technology, Cambridge, MA 02139, USA.

(Received January 16, 1991)

Introduction

When a metal containing elongated inclusions is subjected to a large change of temperature, prismatic dislocations are punched into the matrix to relieve the mismatch stresses induced by the different coefficients of thermal expansion (CTE) of the metal and the inclusion. This phenomenon has been observed in metals reinforced with whiskers (1, 2) and silver chloride containing glass and alumina fibers (3-5). It is also relevant to metals containing submicroscopic second phases of elongated shape, such as S-needles in Al-Cu-Mg, β'-needles in Al-Mg-Si (6) or V(C,N) and AlN rods in steels (7).

Taya and Mori (1) have modelled the longitudinal punching of a row of loops by a mismatching fiber by assuming that the fiber and its plastic zone can be described by two inscribed prolate spheroids. They calculate the potential energy of the system using Eshelby's equivalent inclusion method and find the punching distance by minimizing the sum of the potential energy and the work done by the motion of the loops against the lattice friction stress. Recently, we proposed another model to calculate the number of loops punched by a cylindrical mismatching fiber and the punching distance (8), based on the shear-lag model proposed by Cox (9) and the equilibrium of a row of prismatic loops (10). Experimental data gathered on silver chloride containing glass fibers (5) was in reasonable agreement with the latter model, but less so with that of Ref. (1). We proposed that the observed discrepancy was due to two of the different physical and geometric assumptions made: (i) full plastic relaxation was assumed in Ref. (1) while only partial plastic relaxation was allowed in Ref. (8) and (ii) the fiber and its plastic zone are described in Ref. (1) by inscribed spheroids, while a cylindrical fiber punching a cylindrical row of coaxial loops of same diameter was considered in Ref. (8).

In what follows, we extend Taya and Mori's original model by relaxing condition (i), i.e., by allowing the spheroid fiber to punch more or fewer dislocation loops than the number necessary to exactly relax the longitudinal thermal strain. We also consider the case where radial strains are relaxed independently. We then give examples in two different systems and compare the models to existing data.

Theory

Consider a relaxed fiber of CTE α_f in an infinite, non strain-hardening matrix of CTE α_m. Upon a change of temperature ΔT, the thermal strain α^* is given by:

$$\alpha^* = (\alpha_f - \alpha_m)\,\Delta T \qquad [1]$$

0036-9748/91 $3.00 + .00

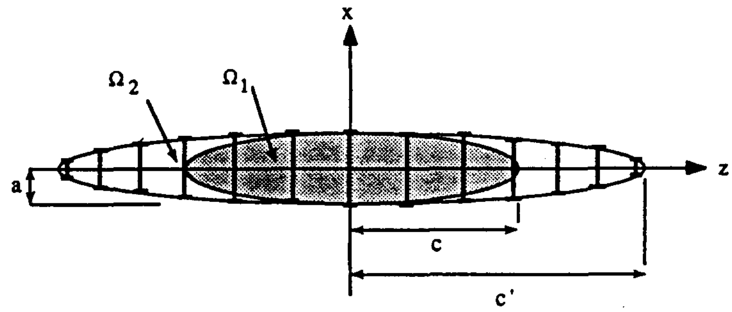

Fig. 1: Spheroidal fiber surrounded by a spheroidal plastic zone of punched prismatic loops (after Ref. 1).

The fiber is represented by a prolate spheroid (Ω_1) of major and minor axes c and a, centred at the origin of the coordinate system $Oxyz$. Prismatic loops are created at the surface of the fiber and are assumed to be punched in the z-direction only. This row of prismatic is described by the surface of another prolate spheroid (Ω_2) inscribing the fiber (Ω_1) and of major and minor axes c' and a (Fig. 1). The prismatic loops are smeared-out and are replaced by the eigenstrains:

$$e_{ij}^{1*} = \begin{pmatrix} \alpha^* & 0 & 0 \\ 0 & \alpha^* & 0 \\ 0 & 0 & (1-\chi)\,\alpha^* \end{pmatrix} \quad [2a]$$

$$e_{ij}^{2*} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \chi\,\alpha^*/\xi \end{pmatrix} \quad [2b]$$

where $\xi = c'/c$ and χ is the degree of prismatic punching in the z direction or degree of plastic relaxation. $\chi = 0$ corresponds to the case where no loops are punched, $\chi = 1$ to that when the mismatch due to the CTE difference in the z-direction is relaxed by prismatic loop punching, i.e., to Taya and Mori's original model. This corresponds to a number N of punched loops of Burgers vector b given by:

$$N\,b = \chi\,c\,\alpha^* \quad [3]$$

The elastic strain energy energy per unit volume U of the system shown in Fig. 1 is given in Ref. (1) as:

$$U = -\frac{f_1}{2}C_{ijkl}\left\{\left(S_{klmn}^{2}e_{mn}^{1**} - e_{kl}^{1**}\right)e_{ij}^{2*} + \left(S_{klmn}^{1}e_{mn}^{1**} - e_{kl}^{1**}\right)e_{ij}^{1*} + \left(S_{klmn}^{2}e_{mn}^{2*} - e_{kl}^{2*}\right)\left(\xi e_{ij}^{2*} + e_{ij}^{1*}\right)\right\} \quad [4]$$

(after correction of two typographical errors in the first term of the right hand-side of equation [13] in Ref. (1)). C_{ijkl}^{f} and C_{ijkl} are the stiffness tensors of the fiber and matrix, S_{klmn}^{1} and S_{klmn}^{2} the fiber and matrix Eshelby's tensors and f_1 the volume fraction of fibers. The only unknown is thus e^{1**} which is determined from Eq. [8] of Ref. (1):

$$C^f_{ijkl}\left(S^2_{klmn}e^{2*}_{mn} - e^{2*}_{kl} + S^1_{klmn}e^{1**}_{mn} - e^{1*}_{kl}\right) = C_{ijkl}\left(S^2_{klmn}e^{2*}_{mn} - e^{2*}_{kl} + S^1_{klmn}e^{1**}_{mn} - e^{1**}_{kl}\right) \quad [5]$$

As in Ref. (1), we use an expression by Tanaka *et al.* (11) for the work per unit volume W spent moving the N dislocations against the lattice friction stress k :

$$W = \chi\, \beta_1\, f_1\, k\, \alpha^*\left(\xi - 1\right) \quad [6]$$

where $\beta_1 = c/a$. The dimensionless punching distance ξ can then be determined by solving

$$\frac{\partial^2 (U + W)}{\partial\xi\, \partial\chi} = 0 \quad [7]$$

In summary, we have expanded Taya and Mori's original model to the case where relaxation differs from that necessary for full relaxation of the fiber in the punching direction. Taya and Mori's original model solved Eq. [7] with the added condition that $\chi=1$.

Results and Discussion

We used a program written in the Mathematica™ symbolic programming language to solve Eqs. [4] and [5], yielding a solution for the potential energy U too long to be reported here. To solve Eq. [7], the value of $(U+W)/f_1$ was minimized numerically using the method of steepest descent from the starting point ($\xi=1$,$\chi=0$), which corresponds physically to the state of the fiber before punching is initiated. The numerical minimisation yields the first minimum of the function investigated and it is thus possible that the absolute minimum is located at another point separated from the local minimum by an energy barrier. In such a case, however, the composite is expected to reach the local minimum and stay in this state, unless the energy barrier can be overcome.

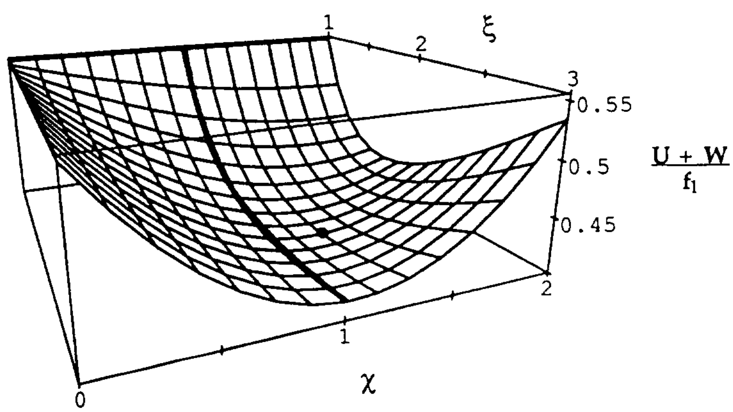

Fig. 2: Graph for $(U+W)/f_1$ as a function of the relative punching distance ξ and the relaxation parameter χ. A dot marks the location of the minimum, (ξ_0, χ_0) and the curve for $\chi=1$ is highlighted. Mismatching SiC spheroid of aspect ratio 3 in a titanium matrix with ΔT=-300 K.

TABLE 1: Thermal and Mechanical Materials Constants.

	G (GPa)	λ (GPa)*	ν	α (10^6/K)	ΔT (K)	k (MPa)
SiC	192.3	99.1	0.17	3.6	—	—
Ti	44	114.5	0.361	8.6	-300	13.7
glass	34.8	42.8	0.22	6.5	—	—
AgCl	5.5	21.1	0.343	30	-100	0.5

* calculated from $\lambda = E\nu / (1+\nu)(1-2\nu)$

To compare Taya and Mori's original model and its modified version presented here, we chose the system Ti/SiC, the properties of which are listed in Table 1. Fig. 2 shows a typical plot of $(U+W)/f_1$ as a function of the two variables ξ and χ. It is apparent that the minimum (ξ_0, χ_0) of this surface is different from the minimum ξ^* of the curve $(U+W)/f_1$ for $\chi=1$, corresponding to Taya and Mori's original model. The value of $(U+W)/f_1$ for $\xi=1$ remains constant, as it should on physical grounds, because this corresponds to no dislocation punching. The parameter χ_0 is larger than 1, meaning that the fiber has punched more dislocation loops than are needed to relax the mismatch α^* in the z-direction alone. This might be because in the absence of lateral plastic relaxation, the fiber is subjected to radial compressive stresses which induce additional longitudinal relaxation. The maximum longitudinal strain due to this Poisson's effect should be of the order of 2ν, so the maximum value of χ_0 should be about $1+2\nu$. This is indeed more than the values we find for χ_0 in the present calculations.

Fig. 3 gives ξ^* and ξ_0 with its corresponding value of χ_0 as a function of the fiber aspect ratio β_1. The general shape of the relative punching distance curves is similar; in particular, both models predict a critical fiber aspect ratio above which punching is suppressed. For small fiber aspect ratios, the present model predicts values of ξ_0 smaller than ξ^* as well as values of χ_0 larger than 1. At large fiber aspect ratios, however, χ_0 becomes smaller than 1 and ξ_0 larger than ξ^*.

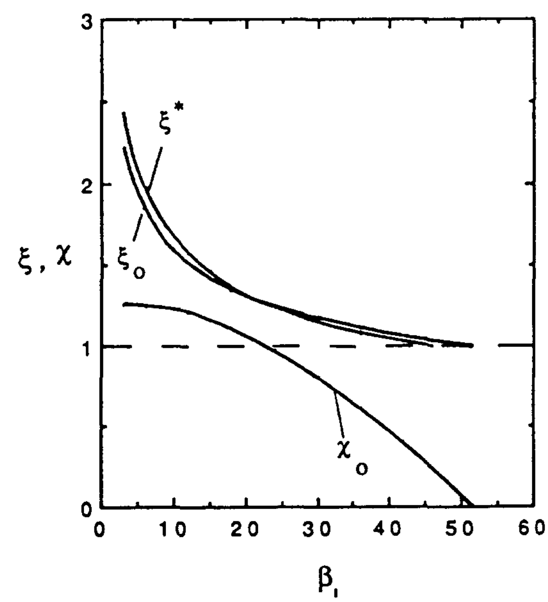

Fig. 3: Punching distance for the system Ti/SiC according to the original model of Ref. (1), ξ^*, and to the modified model, ξ_0, as well as punching parameter, χ_0, as a function of the spheroid aspect ratio β_1.

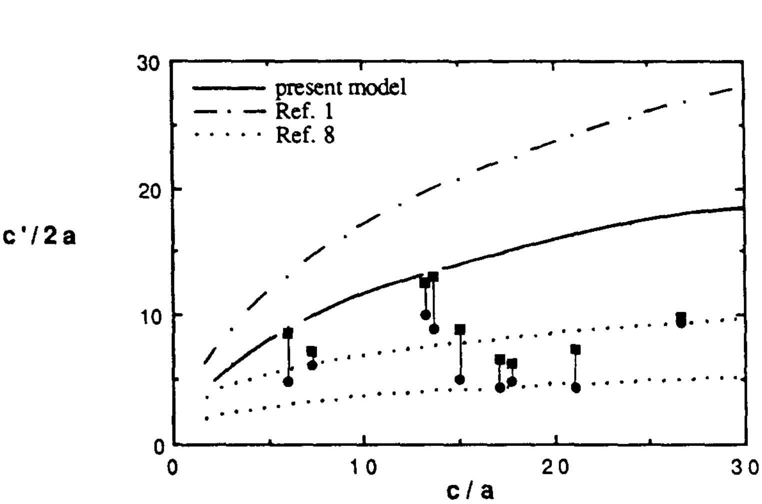

Fig. 4: Dimensionless punching distance for three models and experimental data points from Ref. (5) for the system AgCl/glass.

Curves were also calculated for the system AgCl/glass for which experimental data exist. Fig. 4 shows the calculated dimensionless punching distance $c'/2a$ as a function of the spheroid aspect ratio c/a for both the original and the modified Taya and Mori's models, using the parameters listed in Table 1. Also plotted in the same figure are experimental values reported earlier (5), as well as the prediction range of the model based on the shear-lag assumption and dislocation equilibrium considerations presented in Ref. (8). Unlike Taya and Mori's original model, the modified model presented here is within experimental error of the data. While the quantitative disagreement in punching distance between Taya and Mori's modified model and the shear-lag based model (8) is moderate for the small fiber aspect ratios plotted in Fig. 4, qualitative disagreement between the two models remains since the modified Taya and Mori model predicts a higher dislocation number as well as a critical fiber aspect ratio above which punching is suppressed.

Observations made on the relaxed glass fibers in silver chloride (5) showed that dislocations were punched radially as well as longitudinally, leading to the conclusion that the fibers were radially relaxed. To approximate this situation, we modify e_{ij}^{1*} in Eq. [2a] so that there is no radial mismatch

$$e_{ij}^{1*} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (1-\chi)\,\alpha^* \end{pmatrix} \qquad [8]$$

The resulting curves for $c'/2a$ are shown in Fig. 5 where both the curve for $\chi=1$ and that for variable χ are shifted toward smaller punching distances. We thus conclude that Taya and Mori's models, in both their original and modified form, fit the data on AgCl/glass better if radial relaxation is assumed. The parameter χ was lowered by the assumption of lateral relaxation, however it remained greater than 1 for the range of β_1 in Fig. 5. This indicates that the radial stress does not fully explain why χ can be greater than 1.

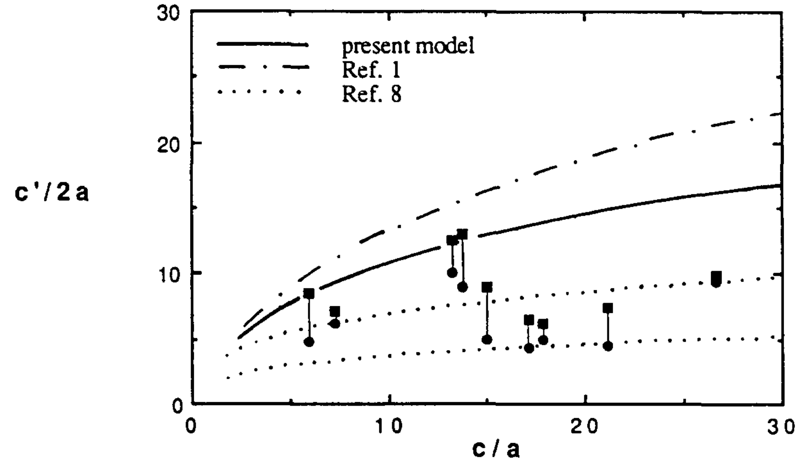

Fig. 5: Punching distance for Taya and Mori's original and modified models assuming no radial mismatch, Eq. [8]. Experimental data points for the system AgCl/glass from Ref. (5).

Conclusions

- A model by Taya and Mori (1) predicting the length of a row of dislocation loops punched from a mismatching prolate spheroid has been extended to the more general case where the degree of plastic relaxation is not set equal to the longitudinal mismatch.
- Calculations made on the systems Ti/SiC and AgCl/glass show that, for small spheroid axis ratios, the modified model predicts a smaller punching distance than the original model. The degree of relaxation can be higher or lower than that predicted by the original model.
- The modified model fits the experimental data on silver chloride containing cylindrical glass fibers of small fiber aspect ratio better than the original model.
- Full lateral relaxation further lowers the punching distance, improving agreement with experiment.

Acknowledgements: D.C.D. gratefully acknowledges a fellowship from the Swiss National Foundation. This work was supported by the National Science Foundation, grant DMR-9002558.

References

1. M. Taya and T. Mori , Acta Metall. 35, 155 (1987).
2. M. Vogelsang, R.J. Arsenault and R.M. Fisher, Metall. Trans. 17A, 379 (1986).
3. M.T. Sprackling, Phil Mag. 13, 1293 (1966).
4. D.C. Dunand and A. Mortensen, "On Plastic Relaxation of Thermal Stresses in Reinforced Metals", Acta Metall. Mat. , in print.
5. D.C. Dunand and A. Mortensen, "Dislocation Emission at Fibers. Part 2: Experiments and Microstructure of Thermal Punching ", submitted to Acta Metall. Mat., 1990.
6. G.C. Weatherly and R.B. Nicholson, Phil. Mag., 17, 801 (1968).
7. M.M.A. Bepari, Metall. Trans. 21A, 2839 (1990).
8. D.C. Dunand and A. Mortensen, "Dislocation Emission at Fibers. Part 1: Theory of Longitudinal Punching by Thermal Stresses", submitted to Acta Metall. Mat., 1990.
9. H.L. Cox, Brit. J. Appl. Phys. 3, 72 (1952).
10. D.C. Dunand and A. Mortensen, "Relaxed Configuration of a Row of Punched Prismatic Dislocation Loops", submitted to Scripta Metall. Mat., 1990.
11. K. Tanaka, K. Narita and T. Mori, Acta Metall. 20, 297 (1972).